

# Solitons and vortices in an evolving Bose-Einstein condensate

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Spatiotemporal evolution of a confined Bose-Einstein condensate is studied by numerically integrating the time-dependent Gross-Pitaevskii equation. Self-interference between the successively expanding and reflecting nonlinear matter waves results in spiral atomic density profile, which subsequently degenerates into an embedding structure: the inner part preserves memory of the initial states while the outer part forms a sequence of necklace-like rings. The phase plot reveals a series of discrete concentric belts. The large gradients between adjacent belts indicate that the ring density notches are dark solitons. In the dynamical process, a scenario of vortex-antivortex pairs are spontaneously created and annihilated, whereas the total vorticity keeps invariant.

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## I. INTRODUCTION

Interference experiments demonstrated that the dilute atomic Bose-Einstein condensate (BEC) is a coherent matter wave.[1, 2, 3] When two spatial separated phase-independent BECs are left to expand, they overlap to form interference fringes. A recent experiment by D.R. Scherer et al[4] revealed that the interference of three independent trapped BECs results in the creation of vortices, which gives definite verification of such nonlinear interference. These experiments and the relevant theoretical studies opened a new arena to explore quantum phenomena at the macroscopic level, as well as nonlinearity effects on the interference and evolution of matter waves.[5, 6] The nonlinearity in BECs arises from the mean-field approximation for short-range interactions between Bose atoms. It accounts for the existence of many coherent nonlinear structures that have been observed in experiments, such as dark,[7, 8, 9] bright[10, 11] and gap solitons,[12] vortices[13, 14, 15] and vortex lattices,[16, 17] etc.

In the BEC, dark soliton (DS) is characterized by a local density minimum and a sharp phase gradient of the wave function at the position of the minimum. Burger et al [7] carried out an experiment to create dark solitons in a strongly elongated condensate by the phase imprinting technique. Theocharis et al [18] introduced the concept of ring dark soliton (RDS) in a 2-dimensional (2D) Bose condensate. It is found that the RDS solutions are unstable to snake instability, whereby they decay into vortex-antivortex (V-AV) pairs. The experimentally observed dynamical instability [8] is due to their quasi-1D character, i.e., a DS stripe becomes unstable against transverse snaking [19, 20, 21].

The tunability of BEC settings allows for rapid change of system parameters and observation of the subsequent quantum dynamics, which can remain coherent for ex-

ceedingly long times because the condensate atoms can be well isolated from the environment. It is also possible to provide means to probe non-equilibrium quantum dynamics of many body systems.[22, 23, 24, 25] In the sudden change limit one can consider that the system is prepared in the ground state of an initial Hamiltonian  $H_i$ , and then evolves under the influence of a final Hamiltonian  $H_f$ . For example, in a one-dimensional optical lattice experiment, the authors found that when the system parameter of BEC in superfluid regime is suddenly changed to the deep Mott insulator regime, i.e.,  $U/t$  large limit, the BEC may revive to the superfluid regime regularly.

It is well-known that a quantized vortex in superfluid cannot simply fade away or disappear, it is only allowed to move out of the condensate or annihilate with another vortex of an opposite circulation. In almost pure condensates, vortices with lifetimes up to tens of seconds have been observed.[15, 16] In this work we report the dynamical evolution of a BEC confined in a 2-dimensional cylinder well. The initial state is prepared by loading the condensate into a narrower harmonic well. By using a topological phase imprinting method we can create one or several vortices in the condensate. The harmonic well is then suddenly changed to a wider cylinder well and the BEC is left to freely expand until it reaches the inner wall of the cylinder well. The reflecting waves then interfere successively with the expanding waves. In a previous publication,[26] we have studied the evolution of the same setup without vortex in the initial state. We demonstrated that ring dark solitons are spontaneously generated and survive for brief time before they are taken place by another set of solitons. In this paper, we further explore the dynamical evolution for the Bose condensate with one or several vortices in the initial state. We find that the condensate first forms a spiral atomic density profile. Subsequently, the system evolve into a peculiar embedding pattern with an inner part that seems to preserve the memory of the initial state. The outer part is a sequence of necklace-like rings. As in the non-vortex case, The notches between these density rings are identi-

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fied as dark solitons, which precess around the inner kernel. The system evolves with co-existence of vortices and dark solitons. A scenario of vortex-antivortex creation and annihilation takes place in the dynamical evolution. But the total vorticity remains invariant and the density distribution preserves the initial symmetry.

## II. THE MODEL

In weak interaction limit, quantum dynamics of the BECs are determined by the nonlinear Gross-Pitaevskii (G-P) equation. We restrict the problem to the 2D plane and employ a cylindrical trap. We model the cylindrical well with  $V(\vec{r}) = V_0[\tanh(\frac{r^2 - r_0^2}{a_0^2}) + 1]$ , where  $r_0$  is the radius of the trap and  $a_0$  is a scaling length parameter relevant to the size of the well. The corresponding characteristic frequency is  $\omega_c = \hbar/ma_0^2$ . When  $V_0$  is large enough, the condensate is completely confined. After re-scaling the parameters by making substitution  $t \rightarrow \omega_c t$ ,  $\vec{r} \rightarrow \vec{r}/a_0$ , and  $\psi \rightarrow \psi/a_0^{3/2}$ , one obtains the reduced dimensionless G-P equation

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + V(\vec{r})\psi + c|\psi|^2\psi, \quad (1)$$

where  $c = 4\pi Na_s/a_0$  and  $a_s$  the s-wave scattering length.

In order to study the quantum dynamics we prepare a steady initial state by employing a narrower harmonic potential  $V_0(\vec{r}) = \alpha r^2/2$ , where  $\alpha$  is an adjustable parameter, and let the wavefunction propagate along the imaginary time of the G-P equation. Quantized vortices are created by using topological phase imprinting technique[15, 27] which is most convenient for rapid preparation of well-defined vortex states. Subsequently, at  $t = 0$ , the harmonic well is lifted and the condensate is allowed to freely expand until it reaches the inner wall of a cylinder well. We investigate the dynamical evolution of the interfering matter waves. The simulation is carried out in a region  $(x, y) \in [-10, 10] \times [10, 10]$  with a refined grid of  $256 \times 256$  nodes, which is sufficient to achieve grid independence. Setting the total time scale as  $t = 15$  and the time step  $\Delta t = 0.00005$ , we numerically integrate the time-dependent G-P equation by using the Crank-Nicolson scheme.[28]

## III. NUMERICAL SIMULATIONS

Figure 1 snapshots the dynamical evolution of a BEC with one vortex at the center of the symmetrical well. In the earlier stages, the condensate freely expands to fill the empty space until it reaches the wall of the cylinder well. Then the reflecting waves and the expanding waves interfere to form concentric density rings. The single vortex always stays at the center. Although vortex is excitation of motion and therefore energetically unstable towards

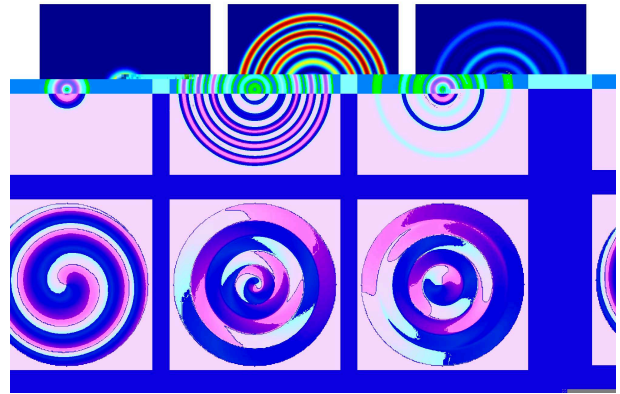


FIG. 1: (Color online) Snapshots of the density (upper panels) and phase distribution (lower panels) in a region  $\{X, Y\} \in [-10, 10]$  (in units of  $a_0$ ). In the initial state there is a vortex at the origin. From left to right:  $t = 0.38$ ,  $t = 5$  and  $t = 10$  (in units of  $\omega_c^{-1}$ ). As  $t \gtrsim 5$ , the phase rings gradually degenerate into discontinuous belts.

relaxation into the ground state which is static, long-range quantum phase coherence regulates the dynamics of quantized vortices.[29] The lower panels of Fig.1 display the corresponding spatial phase distribution of the condensate. As time evolving, the phase begins to skew to form a helix, indicating that while expansion the circular motion of the condensate is not uniform. Novelly, when the evolving time is long enough ( $t > 5$ ), the helix breaks up to form a sequence of discontinuous belts. In the mean time, an inner kernel mimics the initial state is formed, surrounded by a sequence of concentric density rings.

In a previous publication, We have identified that the concentric density notches are ring dark solitons, which survive for a brief time before evolving into another set of dark solitons. The key point for the existence of RDS is the large phase gradients across the density notches. The proof applies for the present case. The details of the demonstration can be found in Ref.[26].

Next we investigate the dynamical evolution of the condensate with four symmetric vortices in the initial state. In this case, the system has a four-fold rotational symmetry. The upper panels of Fig.2 shows the evolution of the BEC density profile at three typical moments  $t = 0.38$ , 5, and 10. When the condensate is released from the initial potential, the four vortices expand outward and gradually disappear due to self-interference. At the same time, four vortices somehow reappear around the central area. For this somehow more complex initial state, the density forms a four-fold symmetrical spiral structure. However, these quasi-1D spiral density arms are unstable towards transverse modulations. As a result, the density arms gradually break up and a peculiar embedding structure forms when  $t \gtrsim 6$  (see left panel in Fig.2). The initial four vortices muster at the central area, surrounded by a sequence of necklace-like density rings. The inner part is quite similar to the initial state. The outer part is again

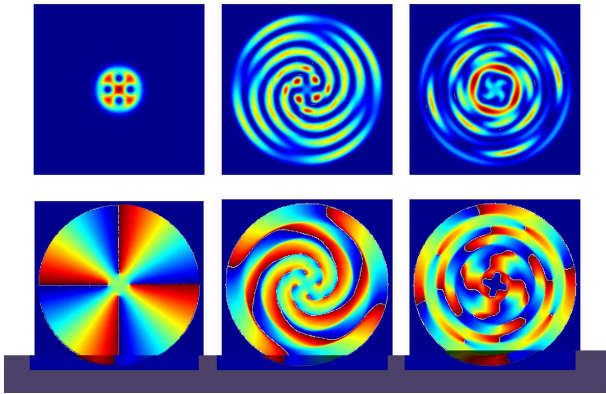


FIG. 2: (Color online) Snapshots of the density (upper panels) and phase distribution (lower panels) at  $t = 0.01$ ,  $t = 5$  and  $t = 10$ . In the initial state there are four symmetric vortices at the center.

the rotated necklace-like dark solitons. After this evolving process, the condensate seems to *throw off* part of its mass to fill the empty space and form ring dark solitons, leaving an inner kernel that preserves the initial momery.

The lower panels of Fig.2 show the corresponding phase plot of the evolving condensate. At the earlier stage, the phase skews into helices. This is because the condensate has nonzero total angular momentum and rotates around the symmetrical axis. But when the evolving time is long enough ( $t \gtrsim 6$ ), the phase plot again shows a sequence of plateau-like belt. Abrupt jumps between adjacent phase belts indicate the formation of solitons. We note that the dynamical soliton can last for a brief period but is still evolving. As estimated in Ref.[26], the duration of the solitons is about 0.5ms for  $c = 20$ .

The instability of the spiral stripes results in rich phenomena of creation and annihilation of V-AV pairs. Figure 3 displays a typical spatial superflow distribution at  $t = 5$ . The evolution of V-AV pairs in the present BEC setting is comparable to the previous work,[20, 21, 30] where several separated BECs were allowed to expand and merge together. Although the creation and annihilation of the V-AV pairs is rather complex, the whole distribution keeps a perfect four-fold symmetry. After a long time, this V-AV scenario gradually fades away when the density profile wears the concentric necklace-like solitons.

Figure 4 takes snapshots of the density and phase distribution with seven vortices in a six-fold symmetric initial state, in which one vortex locates at the origin surrounded by the other six vortices. Analogous to the case of four vortices, six spiral density arms are first derived from the condensate. Then these arms are broken by continuous interference and oscillations. An embedding structure again appears. The inner kernel resembles the

initial state. In the course of dynamical evolution, the density distribution always exhibits a six-fold symmetry and the total vorticity keeps invariant.

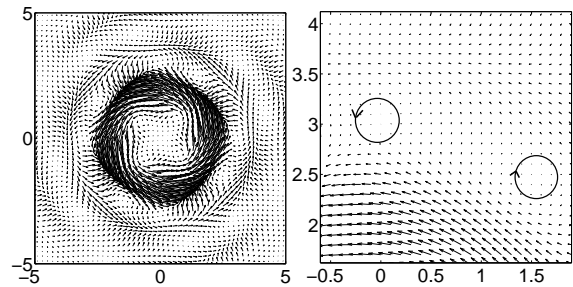


FIG. 3: Vectorial plot of the super-flow with four vortices at  $t = 5$ . The right panel is an enlarged local regime. The arrowed circles indicate the positions of a vortex and an antivortex.

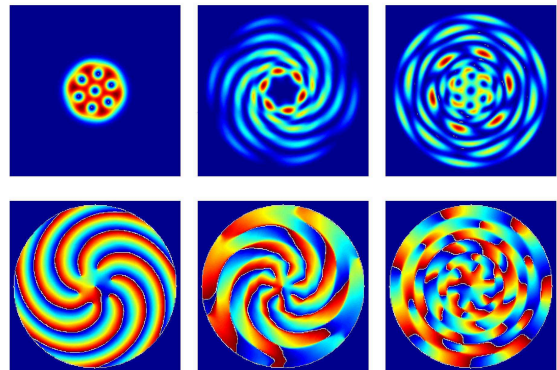


FIG. 4: (Color online) Same as in Fig.1 for BEC with seven symmetric vortices at the center. The snapshot times from left to right are  $t = 0.38$ ,  $t = 5$  and  $t = 10$ .

#### IV. SUMMARY

We have studied the dynamical evolution of a repulsive Bose-Einstein condensate in a 2D external potential. We found that the density profile shows an embedding structure in which the inner kernel resemble the initial state while the outer part forms a sequence of necklace-like solitons, which circulates around the inner kernel. These solitons survives for a brief time before evolving into another set of solitons. A scenario of creation and annihilation of vortex-antivortex pairs takes place in the evolving process.

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